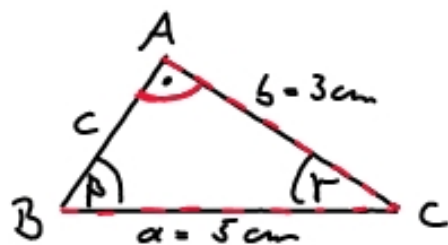


S 227

III a)



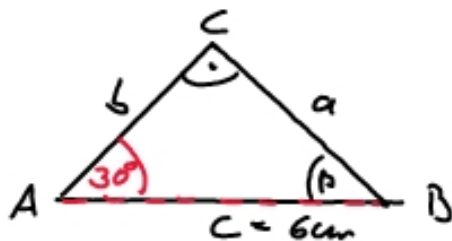
$$\sin(\beta) = \frac{b}{a} = \frac{3}{5}$$

$$\beta = \arcsin\left(\frac{3}{5}\right) = 36,87^\circ$$

$$\cos(\beta) = \frac{c}{a} = \frac{4}{5}$$

$$\beta = \arccos\left(\frac{4}{5}\right) = 36,87^\circ$$

4)



$$\beta = (180^\circ - 90^\circ - 30^\circ) = 60^\circ$$

$$\sin(30^\circ) = \frac{a}{6} \Leftrightarrow a = 6 \cdot \sin(30^\circ) = 3$$

$$\cos(30^\circ) = \frac{b}{6} \Leftrightarrow b = 6 \cdot \cos(30^\circ) = 5,2$$

$$a^2 = b^2 + c^2$$

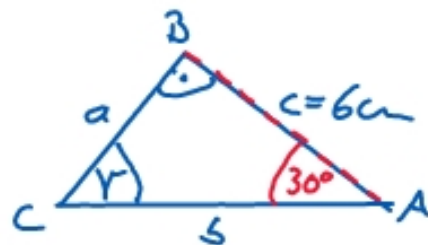
$$25 = 9 + c^2 \quad | -9$$

$$16 = c^2 \quad | \sqrt{\quad}$$

$$c = \pm \sqrt{16} = \pm 4 \quad \Rightarrow \{c = 4\}$$

$$\sin(\gamma) = \frac{c}{a} = \frac{4}{5}$$

$$\gamma = \arcsin\left(\frac{4}{5}\right) = 53,13^\circ$$

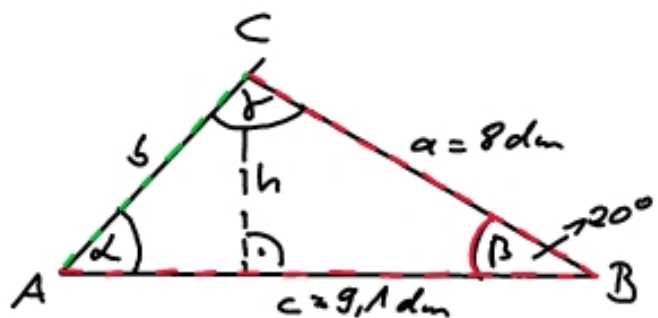


$$\gamma = 60^\circ$$

$$\cos(30^\circ) = \frac{b}{6} \Leftrightarrow b = \frac{6}{\cos(30^\circ)} = 6,93$$

$$\sin(30^\circ) = \frac{a}{6,93} \Leftrightarrow a = 6,93 \cdot \sin(30^\circ) \\ a = 3,46$$

IV



Fläche. $A = \frac{1}{2} \cdot c \cdot h$

$$\sin(\beta) = \frac{h}{a}$$

$$h = a \cdot \sin(\beta)$$

$$h = 8 \cdot \sin(20^\circ) = 2,74$$

$$A = \frac{1}{2} \cdot 9,1 \cdot 2,74 = 12,47 \text{ dm}^2$$

$$U = 8 + 9,1 + 3,16 = 20,26 \text{ dm}$$

cos: $b^2 = a^2 + c^2 - 2ac \cdot \cos(\beta)$

$$b^2 = 8^2 + 9,1^2 - 2 \cdot 8 \cdot 9,1 \cdot \cos(20^\circ)$$

$$b^2 = 10 \quad \Rightarrow \quad b = \sqrt{10} = \underline{3,16}$$

sin: $\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} \quad \Leftrightarrow \quad \sin(\alpha) = \frac{a}{b} \cdot \sin(\beta)$

$$\sin(\alpha) = \frac{8}{3,16} \cdot \sin(20^\circ) = 0,865$$

$$\alpha = \arcsin(0,865) = 60^\circ$$

Σ: $\gamma = 180 - (60^\circ + 20^\circ) = 100^\circ$

Vektoren



$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{pmatrix} = (a_1, a_2, \dots, a_n)^T$$

$$\vec{x} \in \mathbb{R}^3 \rightarrow \vec{x} = (x_1, x_2, x_3)^T \quad \begin{array}{l} \text{Vektor} \\ \alpha \in \mathbb{R} \quad \text{Skalar} \end{array}$$

$$\alpha \cdot \vec{x} = \alpha \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \alpha \cdot x_1 \\ \alpha \cdot x_2 \\ \alpha \cdot x_3 \end{pmatrix}$$

$$\begin{pmatrix} -4 \\ 6 \\ -8 \end{pmatrix} = -2 \cdot \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \quad \begin{array}{l} \text{"quasi gleich"} \\ \text{linear abhängig} \end{array}$$



$$\vec{x} + \vec{y} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \\ 5 \end{pmatrix}$$

Skalarprodukt \Rightarrow Lösung ist ein Skalar

$$\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} = 1 \cdot 2 + (-2) \cdot 1 + 3 \cdot 4 = 12$$

Vektorprodukt \Rightarrow Lösung ist ein Vektor

$$\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 3 \\ -2 & 1 & 1 \\ 3 & 4 & 2 \\ 1 & 2 & 2 \\ -2 & 1 & 1 \\ 3 & 4 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \cdot 4 & -1 \cdot 3 \\ 3 \cdot 2 & -4 \cdot 1 \\ 1 \cdot 1 & -2 \cdot (-2) \end{pmatrix} = \begin{pmatrix} -11 \\ 2 \\ 5 \end{pmatrix} = \vec{n}$$

↓
Stellungsvektor

$$2. \left[\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} \right] * \left(3 \cdot \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} - 2 \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \right)$$

$$\begin{pmatrix} 1 & & 4 \\ -2 & & 1 \\ 3 & & -1 \\ 1 & & 4 \\ -2 & & 1 \\ 3 & & -1 \end{pmatrix} = \begin{pmatrix} 2 & - & 3 \\ 12 & - & (-1) \\ 1 & - & (-8) \end{pmatrix} = \begin{pmatrix} -1 \\ 13 \\ 9 \end{pmatrix}$$

$$2. \begin{pmatrix} -1 \\ 13 \\ 9 \end{pmatrix} * \begin{pmatrix} 3 & -4 \\ 6 & -2 \\ 3 & -8 \end{pmatrix} = \begin{pmatrix} -2 \\ 26 \\ 18 \end{pmatrix} * \begin{pmatrix} -1 \\ 4 \\ -5 \end{pmatrix} = 2 + 104 - 90 = 16$$

Lineare (Un) Abhängigkeit

Linearkombination: $\alpha \cdot \vec{a} + \beta \cdot \vec{b} + \gamma \cdot \vec{c} = \vec{0}$

$\alpha = \beta = \gamma = 0 \rightarrow$ Trivialsolution \Rightarrow linear unabhängig

$\vec{a} = (1; -2; 1)$; $\vec{b} = (-1; 3; 2)$; $\vec{c} = (-2; 1; -3)$

$$\left| \begin{array}{ccc|c} \alpha & -\beta & -2\gamma & = 0 \\ -2\alpha & +3\beta & +\gamma & = 0 \\ \alpha & +2\beta & -3\gamma & = 0 \end{array} \right| \begin{array}{l} \cdot 2 \\ \cdot (-1) \end{array}$$

lineare Unabhängigkeit

$$\left| \begin{array}{ccc|c} \alpha & -\beta & -2\gamma & = 0 \\ 0 & \beta & -3\gamma & = 0 \\ 0 & 3\beta & -\gamma & = 0 \end{array} \right| \begin{array}{l} \cdot 3 \\ \cdot (-3) \end{array}$$
$$\left| \begin{array}{ccc|c} \alpha & -\beta & -2\gamma & = 0 \\ 0 & \beta & -3\gamma & = 0 \\ 0 & 0 & 8\gamma & = 0 \end{array} \right|$$

$\gamma = \beta = \alpha = 0 \Rightarrow$ Trivialsolution

237

$$1) \alpha \cdot \vec{a} + \beta \cdot \vec{b} + \gamma \cdot \vec{c} = \vec{0}$$

$$\left| \begin{array}{ccc|c} 2\alpha + 5\beta - 2\gamma = 0 & 2 \cdot (-\gamma) + 5 \cdot \gamma - 2\gamma = \gamma = 0 & & \\ \alpha - \beta + 2\gamma = 0 & -\gamma - \beta + 2\gamma = \gamma - \beta = 0 \Leftrightarrow \beta = \gamma & & \\ 3\alpha + 3\gamma = 0 & \rightarrow 3\alpha + 3\gamma = 0 \Leftrightarrow 3\alpha = -3\gamma \Leftrightarrow \alpha = -\gamma. & & \end{array} \right.$$

$$\Rightarrow \alpha = \beta = \gamma = 0$$

Trivialsolution

$$\text{Pivot} \left| \begin{array}{ccc|c} 2\alpha + 5\beta - 2\gamma = 6 & & & \\ \alpha - \beta + 2\gamma = 5 & & & \\ 3\alpha + 3\gamma = -6 & & & \end{array} \right| \begin{array}{l} \cdot (-2) \\ \cdot (-3) \end{array}$$

$$\alpha = -47$$

$$\beta = 38$$

$$\gamma = 45$$

$$\text{Pivot} \left(\begin{array}{ccc|c} \alpha - \beta + 2\gamma = 5 & & & \\ 0 & 7\beta - 6\gamma = -4 & & \\ 0 & 3\beta - 3\gamma = -21 & & \end{array} \right) \cdot (-\frac{7}{3})$$

$$\left| \begin{array}{ccc|c} \alpha - \beta + 2\gamma = 5 & & & \\ 0 & \beta - \gamma = -7 & & \\ 0 & 0 & \gamma = 45 & \end{array} \right|$$