S 165

1) $6 \cdot \ln \sqrt[3]{e^2} - \frac{8}{e^{2\ln 0{,}5}} - \left(\frac{1}{2} e^{\ln 3^2} - \ln \frac{1}{\sqrt{e}}\right) + \frac{8}{\ln e^2} + e^{2 \ln 3}$

$\ln\left(e^{2/3}\right)^6 - 8 e^{\ln (1/2)^{-2}} - \left(\frac{1}{2} \cdot 3^2 - \ln e^{-1/2}\right) + \frac{8}{2} + e^{\ln 3^2}$

$4 \quad - 32 \quad - \left(\frac{9}{2} + \frac{1}{2}\right) + 4 + 9 = -20$

2) $3 \ln e^5 - 2 \cdot \left(e^{2\ln 2} + \ln \frac{1}{\sqrt[4]{e}}\right) + \frac{10}{e^{\ln \sqrt{4}}} + 0{,}5 \cdot e^{\ln 3}$

$\ln\left(e^5\right)^3 - 2 \cdot \left(e^{\ln 2^2} + \ln e^{-1/4}\right) + \frac{10}{\sqrt{4}} + \frac{1}{2} \cdot 3$

$15 - 2 \cdot \left(4 - \frac{1}{4}\right) + 5 + \frac{3}{2} = 14$

3) $\frac{1}{6} \cdot \operatorname{ld} 2^3 + 3 \cdot e^{2\ln 0{,}5} - \lg \sqrt{10} + 4 \cdot \left(2^{4 \operatorname{ld} \frac{1}{2}} - 8 \cdot \ln \frac{1}{\sqrt{e}}\right) - 4 \cdot 10^{\frac{1}{4} \lg 256}$

$\frac{1}{6} \cdot 3 + 3 \cdot e^{\ln (1/2)^2} - \lg 10^{1/2} + 4 \cdot \left(2^{\operatorname{ld} (1/2)^4} - 8 \cdot \ln e^{-1/2}\right) - 4 \cdot 10^{\lg (256)^{1/4}}$

$\frac{1}{2} + 3 \cdot \frac{1}{4} - \frac{1}{2} + 4 \cdot \left(\frac{1}{16} - 8\left(-\frac{1}{2}\right)\right) - 4 \cdot 4$

$\frac{1}{2} + \frac{3}{4} - \frac{1}{2} + \frac{1}{4} + 16 - 16 = 1$

4) $\frac{2}{3}\cdot\left(\log 1000 - \frac{1}{2}\right) - \frac{2}{e^{\ln 0,5}} + 2^{3+\mathrm{ld}\,3} - \left(10^2\right)^{\log 3} + \ln\left(\frac{1}{\sqrt[3]{e}}\right)^2 - 4\,\mathrm{ld}\sqrt{2}$

$\frac{2}{3}\cdot\left(\log 10^3 - \frac{1}{2}\right) - \frac{2}{\frac{1}{2}} + 2^3\cdot 2^{\mathrm{ld}\,3} - 10^{\log 3^2} + \ln e^{-\frac{2}{3}} - \mathrm{ld}\left(2^{\frac{1}{2}}\right)^4$

$2 - \frac{1}{3} - 4 + 8\cdot 3 - 9 - \frac{2}{3} - 2 = 10$

S169

I.1) $3\cdot\log x - 4\cdot\log \frac{2}{x} - \frac{1}{3}\cdot\log\left(x^2\right)^6 = \frac{2}{3}\lg 27 + \frac{1}{2}\lg x^4 - 2\log 6$

$\lg x^3 - \lg\left(\frac{2}{x}\right)^4 - \lg\left(x^{12}\right)^{\frac{1}{3}} = \lg 27^{\frac{2}{3}} + \lg\left(x^4\right)^{\frac{1}{2}} - \lg 6^2$

$\lg\frac{x^3}{2^4/\cancel{x^4}\cdot\cancel{x^4}} = \lg\frac{3^2\,x^2}{6^2} \qquad \mid\ \uparrow 10^x$

$\frac{1}{16}x^3 = \frac{\overset{1}{\cancel{9}}}{\underset{4}{\cancel{36}}}\,x^2 \qquad \mid : x^2\cdot 16$

$x = 4 \qquad \mathbb{D} = x\in\mathbb{R}^{>0}$

$\Rightarrow \mathbb{L} = \{4\}$

S 169

I.2) $3\cdot\ln 4 - 0{,}5\ln {}^{16}/_{x^4} + 2\ln 8 = 1{,}5\ln x^4 - 8\ln\sqrt[4]{1/x} - 2\cdot\ln {}^1/_4$

$$\ln 4^3 - \ln(^{16}/_{x^4})^{1/2} + \ln 8^2 = \ln(x^4)^{3/2} - \ln(x^{-1/4})^8 - \ln(1/4)^2$$

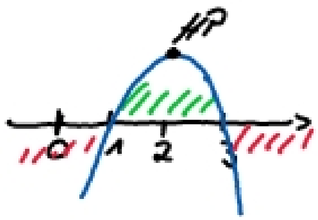
$$\ln\frac{4^3\cdot 8^2}{4/x^2} = \ln\frac{x^6}{x^{-2}\,1/4^2} \quad | \uparrow e^x$$

$$4^2\cdot 8^2\cdot x^2 = 4^2\cdot x^8 \quad | : 4^2 : x^2$$

$$\frac{4^2\cdot 8^2}{4^2} = x^6 \iff 8^2 = (2^3)^2 = 2^6 = x^6$$

$x = 2 \qquad \mathbb{D} = x\in\mathbb{R}^{>0} \qquad \mathbb{L} = \{2\}$

II. 3) $f(x) = {}^2/_5\cdot\ln(4x-3-x^2) = {}^2/_5\cdot\ln(-(x^2-4x+3))$
$= {}^2/_5\cdot\ln(-\underbrace{(x-3)(x-1)})$



$x = 2: -(2-3)(2-1) > 0$

$x = 0: -(0-3)(0-1) < 0$

$x = 4: -(4-3)(4-1) < 0$

$\lim\limits_{x\to 1^+} f(x) = [{}^2/_5\cdot\ln(0^+)] = -\infty$

$f(2) = {}^2/_5\cdot\ln(1) = 0$

$\mathbb{W} = y\in\mathbb{R}^{\le 0}$

$\mathbb{D} = x\in\,]1;3[\, = x\in(1;3)$

Ableitungen

$f(x) = a \cdot x^n$

$f'(x) = a \cdot n \cdot x^{n-1}$

$f(x) = 5 \cdot x^4$

$f'(x) = 5 \cdot 4 \cdot x^{4-1} = 20 \cdot x^3$

$f(x) = 6 \cdot \sqrt[3]{x^2} = 6 \cdot x^{2/3}$

$f'(x) = 6 \cdot \frac{2}{3} x^{2/3 - 1} = 4 \cdot x^{-1/3} = 4 \cdot \frac{1}{\sqrt[3]{x}}$

Potenzklasse

$f(x) = \heartsuit^n \rightarrow f'(x) = n \cdot \heartsuit^{n-1} \cdot \heartsuit'$

$f(x) = (2x-5)^3 \rightarrow f'(x) = 3 \cdot (2x-5)^{3-1} \cdot (2x-5)'$

$= 6 \cdot (2x-5)^2$

Exponential

$$f(x) = e^{\heartsuit} \rightarrow f'(x) = e^{\heartsuit} \cdot \heartsuit'$$

$$f(x) = e^{5-2x^2} \rightarrow f'(x) = e^{5-2x^2} \cdot (5-2x^2)' = -4x \cdot e^{5-2x^2}$$

Trigonometrie

$$f(x) = \sin(\heartsuit) \rightarrow f'(x) = \cos(\heartsuit) \cdot \heartsuit'$$

$$= \cos(\heartsuit) \qquad\qquad = -\sin(\heartsuit) \cdot \heartsuit'$$

Logarithmus

$$f(x) = \ln(\heartsuit) \rightarrow f'(x) = \frac{1}{\heartsuit} \cdot \heartsuit'$$

$$f(x) = \ln(2x-7) \rightarrow f'(x) = \frac{1}{2x-7} \cdot 2$$

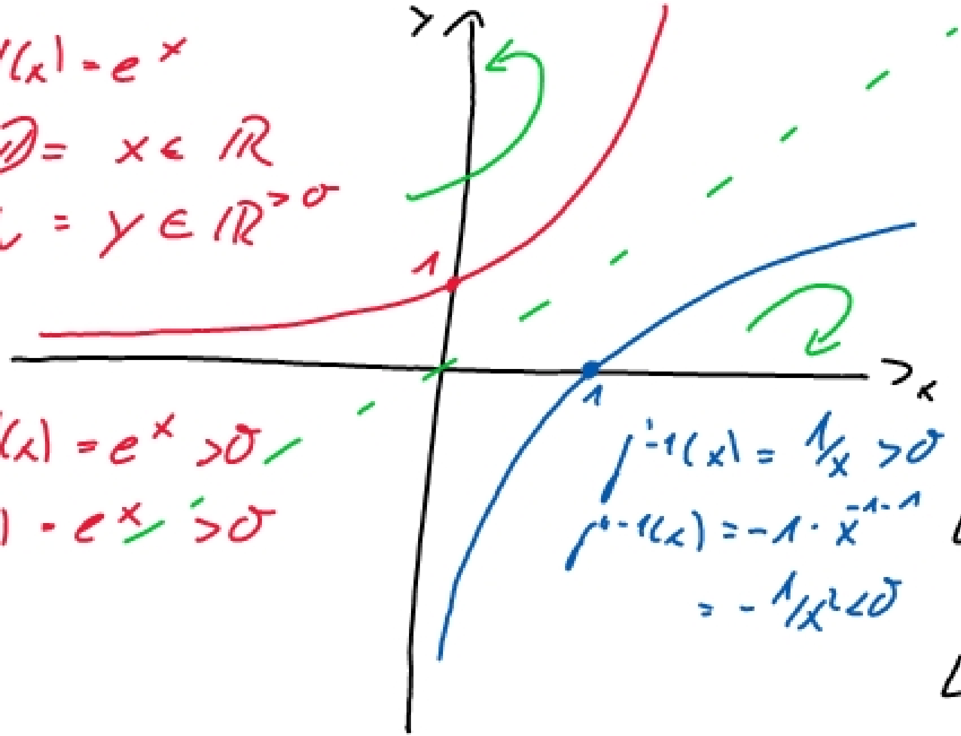
$f(x) = e^x$

$\mathbb{D} = x \in \mathbb{R}$

$\mathbb{W} = y \in \mathbb{R}^{>0}$

$f'(x) = e^x > 0$

$f''(x) = e^x > 0$



$f^{-1\prime}(x) = 1/x > 0$

$f^{-1\prime\prime}(x) = -1 \cdot x^{-1-1}$

$= -1/x^2 < 0$

$f^{-1}(x) = \ln(x)$

$\mathbb{D} = x \in \mathbb{R}^{>0}$

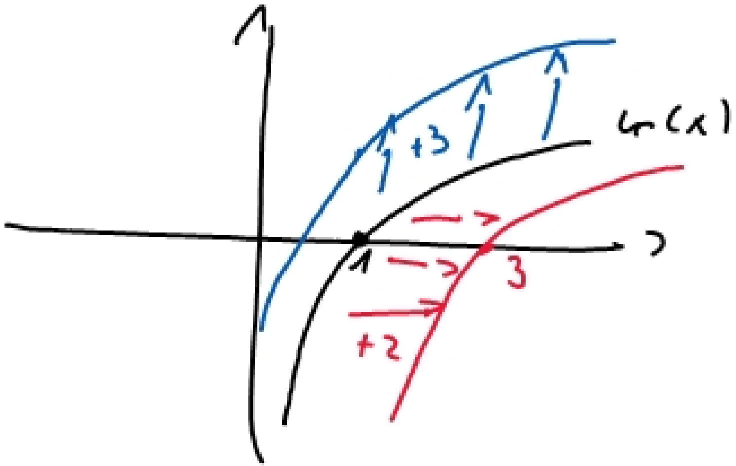
$\mathbb{W} = y \in \mathbb{R}$

$\ln 0 = y$

$e^y = 0$ ↯

$\ln(<0) = y$

$e^y < 0$ ↯



$f(x) = \ln(x) + 3$

$f(x) = \ln(x-2)$