S 147 Nr. 2

$$\left(\begin{array}{cccc|c} -1 & -1 & 2 & 2 & -4 \\ 2 & 1 & -3 & -1 & -2 \\ 3 & -2 & -1 & -1 & 6 \\ -1 & -3 & 2 & 3 & -3 \end{array}\right) \quad | \cdot 2 \quad | \cdot 3 \quad (-1)$$

$$\left(\begin{array}{cccc|c} -1 & -1 & 2 & 2 & -4 \\ 0 & -1 & 1 & 3 & -10 \\ 0 & -5 & 5 & 5 & -6 \\ 0 & -2 & 0 & 1 & 1 \end{array}\right) \quad | \cdot (-5) \quad | \cdot (-2)$$

$$\left(\begin{array}{cccc|c} -1 & -1 & 2 & 2 & -4 \\ 0 & -1 & 1 & 3 & -10 \\ 0 & 0 & 0 & -10 & 44 \\ 0 & 0 & -2 & -5 & 21 \end{array}\right)$$

$x_1 = -1{,}1$

$x_2 = -2{,}7$

$x_4 = -4{,}4$

$x_3 = 0{,}5$

Beispiel : S 147 Nr. 1

$$\left(\begin{array}{ccc|c} -2 & 3 & -1 & -1 \\ 4 & -5 & 2 & 4 \\ 2 & -5 & 6 & 2 \end{array}\right) \quad \begin{array}{c} 1\cdot 2 \\ + \\ 1 \\ + \end{array}$$

$$\cancel{\left(\begin{array}{ccc|c} -2 & 3 & -1 & -1 \\ 0 & & & \\ 0 & & & \end{array}\right)}$$

$$\left(\begin{array}{ccc|c} -2 & 3 & -1 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & 5 & 1 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} -2 & 3 & -1 & -1 \\ 0 & -2 & 5 & 1 \\ 0 & 0 & 0 & 2 \end{array}\right)$$

$A_{3,3}$ A b

det -> $10 \neq 0$

$Rg(A) = 3$

$Rg(A|b) = 3$

$Rg(A) = 2$, da det $= 4 \neq 0$

$Rg(A|b) = 3$, da det $= 8 \neq 0$ $\Big\}\neq$

SS 2008

$$\left(\begin{array}{l} x + 5y + 2z = 4 \\ -x + y - z = 0 \\ 2x - 3y - 2z = 7 \end{array}\right)$$

a) Rangkriterium ; b) [Cramer]

$$\left|\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & -1 & -4 & 7 \\ 12 & 0 & -23 & 46 \end{array}\right|$$

$$\underbrace{\boxed{\begin{matrix} -1 & 1 & -1 \\ 0 & -1 & -4 \\ 12 & 0 & -23 \end{matrix}}}_{A} \qquad \underbrace{\begin{matrix} 0 \\ 7 \\ 46 \end{matrix}}_{b}$$

$$\vec{x} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$$

$\det(A) = -23 \neq 0 \Rightarrow Rg(A) = 3$ Maximalrang

$=$

$\det(A|b) = -23 \neq 0 \Rightarrow Rg(A|b) = 3$

S 153 Nr. 2

$$\left(\begin{array}{ccc|c} -1 & -1 & -2 & 1 \\ 1 & 2 & 3 & 2 \\ 3 & 1 & \alpha & \beta \end{array}\right) \quad + \quad |\cdot 3 \quad +$$

$$\left(\begin{array}{ccc|c} -1 & -1 & -2 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & -2 & \alpha - 6 & \beta + 3 \end{array}\right) \quad |\cdot 2 \quad +$$

$$\left(\begin{array}{ccc|c} -1 & -1 & -2 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & \underset{0}{\alpha - 4} & \underset{0}{\beta + 9} \end{array}\right)$$

1. $\alpha = 4 \wedge \beta = -9 \Rightarrow$ unendlich viele Lösungen

$\det(A) = -1 = \det(A|b) \Rightarrow Rg(A) = 2 = Rg(A|b)$

2. $\alpha = 4 \wedge \beta \neq -9 \Rightarrow$ keine Lösung

$$\left.\begin{array}{l} \det(A) = \begin{vmatrix} -1 & -1 \\ 0 & 1 \end{vmatrix} = -1 \\ \det(A|b) = \begin{vmatrix} -1 & -1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & \beta+9 \end{vmatrix} = -\beta - 9 \neq 0 \end{array}\right\} \begin{array}{c} Rg(A) = 2 \\ \neq \\ Rg(A|b) = 3 \end{array}$$

3. $\alpha \neq 4$

$\Downarrow$

eine Lösung

$$\det(A) = \begin{vmatrix} -1 & -1 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & \alpha-4 \end{vmatrix} = -\alpha + 4 \neq 0 = \det(A|b)$$

$$Rg(A) = 3 = Rg(A|b)$$

$\alpha = 4 \ \wedge \ \beta = -9$:

$$\left(\begin{array}{ccc|c} -1 & -1 & -2 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

frei wählbare Variable : $x_3 = \gamma$

$$\begin{aligned} -x_1 - x_2 - 2\gamma &= 1 \\ x_2 + \gamma &= 3 \quad \Rightarrow \quad x_2 = 3 - \gamma \end{aligned}$$

$$-x_1 - (3-\gamma) - 2\gamma = 1 \quad \Rightarrow \quad x_1 = -\gamma - 4$$

$$\vec{x} = \begin{pmatrix} -4 - \gamma \\ 3 - \gamma \\ 0 + \gamma \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$